Spin light of neutrino in gravitational fields

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Abstract

We predict a new mechanism for the spin light of neutrino ($SL\nu$) that can be emitted by a neutrino moving in gravitational fields. This effect is studied on the basis of the quasiclassical equation for the neutrino spin evolution in a gravitational field. It is shown that the gravitational field of a rotating object, in the weak-field limit, can be considered as an axial vector external field which induces the neutrino spin procession. The corresponding probability of the neutrino spin oscillations in the gravitational field has been derived for the first time. The considered in this paper $SL\nu$ can be produced in the neutrino spin-flip transitions in gravitational fields. It is shown that the total power of this radiation is proportional to the neutrino gamma factor to the fourth power, and the emitted photon energy, for the case of an ultra relativistic neutrino, could span up to gamma-rays. We investigate the $SL\nu$ caused by both gravitational and electromagnetic fields, also accounting for effects of arbitrary moving and polarized matter, in various astrophysical environments. In particular, we discuss the $SL\nu$ emitted by a neutrino moving in the vicinity of a rotating neutron star, black hole surrounded by dense matter, as well as by a neutrino propagating in the relativistic jet from a quasar.

1 Introduction

It is well known that a neutrino with non-zero mass has non-trivial electromagnetic properties [1] (for the recent study of the electromagnetic properties of a massive neutrino

see [2, 3]). In particular, the Dirac massive neutrino possesses non-vanishing magnetic moment. Even the massive Majorana neutrino can have transitional magnetic moment.

In a series of our papers we have developed the Lorentz invariant approach to neutrino spin (and also flavour [4,5]) evolution in different environments accounting for the presence of not only electromagnetic fields [6,7] and weakly interacting with neutrino matter [8] but also for other types of neutrino non-derivative interactions [9]. On this basis we have also predicted [10] the possibility for a new mechanism of electromagnetic radiation by neutrino moving in background matter and/or electromagnetic fields. The new mechanism of electromagnetic radiation (we have named [10] this radiation *"spin light of neutrino"* ($SL\nu$)) originates from the neutrino spin precession that can be produced whether by weak interactions with matter or by electromagnetic interactions with external electromagnetic fields. The latter possibility was also considered before in [11]. A review and some new results on our studies of neutrino oscillations in matter and external fields is given in [12].

As it is shown in [10], the total power of the $SL\nu$ is not washed out even when the emitted photon refractive index in the background matter is equal to unit. That is why the $SL\nu$ can not be considered as the neutrino Cherenkov radiation (see, for example, [13] and references therein). The total power of the $SL\nu$ in matter is proportional to the fourth power of the matter density and the neutrino Lorentz factor. The $SL\nu$ is strongly beamed in the direction of neutrino propagation and is confined within a small cone given by $\vartheta \sim \gamma^{-1}$. The energy range of emitted $SL\nu$ photons, for the case of the relativistic neutrino, could span up to gamma-rays. These properties of $SL\nu$ enables us to predict that this radiation should be important in different astrophysical environments (quasars, gamma-ray bursts etc) and in dense plasma of the early Universe.

In this paper we should like to introduce a new type of the $SL\nu$ that could originate from the neutrino spin precession induced by a gravitational field in different astrophysical and cosmological settings. This study is based on consideration of the neutrino spin evolution problem in gravitational fields. We also consider the neutrino spin oscillations in gravitational fields and derive for the first time the corresponding probability.

The problem of the neutrino flavour oscillations and neutrino spin evolution in the presence of gravitational fields have been studied in many papers. The gravitational effect on flavour neutrinos from collapsing stars was considered in [14]. Neutrino spin procession and oscillations in gravitational fields were also studied in [15–20]. The important obtained result of these studies is that if the contribution of gravitational interaction is diagonal in spin space (that is the case of gravitational fields of non-rotating massive objects), then gravity cannot produce the neutrino spin procession on its own. As it is pointed out in [16], the off-diagonal terms in neutrino spin space could appear from the interaction of a neutrino magnetic moment with a magnetic field.

A rather detailed discussion on various aspects of the neutrino spin and chiral dynamics in the presence of gravitational fields can be found in the recent papers [21–25] where, in particular, the case of a strong field in a Schwarzschild space-time background have been also considered. The case of a weak field was also studied in [18]. However, the possibility of the electromagnetic radiation due to the neutrino spin procession in a gravitational field has never been discussed before.

Here below we apply the Lorentz invariant approach, based on the generalization of the quasiclassical Bargmann-Michel-Telegdi equation for the case of different non-derivative interactions of neutrino with external fields [6, 8, 9], to the study of the neutrino spin evolution in weak gravitational fields. We found out that weak gravitational fields of a rotating source enters the neutrino spin evolution equation as an axial-vector field, that, as it has been shown in [9], produces the spin precession. Then, using the main idea of [10] that a neutrino with spin processing in the external environment should radiate electromagnetic waves, we investigate this spin light of neutrino in gravitational fields in the vicinity of the rotating neutron star, the accretion disk of a rotating black hole and in the relativistic jet of a quasar.

2 Neutrino spin evolution equation in gravitational fields

Following the study of ref. [18], our starting point is the Dirac equation for a neutrino in a curved space-time [26]:

$$[i\gamma^\mu(x)D_\mu - m]\Psi = 0, \tag{1}$$

where $D_\mu = \partial_\mu + \Gamma_\mu$ is the covariant derivative operator, $\Gamma_\mu = -(i/8)e^\nu_a e_{\nu b;\mu}[\gamma^a, \gamma^b]$ is the spin connection, and e^μ_a are the tetrads. Latin indices refer to a local Minkowski frame, while greek indices refer to general curvilinear coordinates. The gamma matrices $\gamma^\mu = \gamma^a e^\mu_a$ satisfy the conditions $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$ and $D_\mu\gamma^\nu = 0$. The metric tensor $g_{\mu\nu}$ and the tetrads e^μ_a are related by

$$g_{\mu\nu} = \eta_{ab}e^a_\mu e^b_\nu, \tag{2}$$

where η_{ab} is the Minkowski metric.

We consider below the limit of a weak gravitational field which is appropriate for the most of astrophysical systems. Indeed, we are also restricted to the weak-field limit by our Lorentz invariant approach in description of the neutrino spin evolution on the basis of the generalized Bargmann-Michel-Telegdi equation. Therefore, we perform our analysis within the linearized theory of gravity by assuming that the metric can be written as

$$g^{\mu\nu} = \eta^{\mu\nu} + 2kh^{\mu\nu}, \tag{3}$$

and treat the second term in the right-hand side as a perturbation over the flat Minkowski space. In this approach $h^{\mu\nu}$ is the gravitation field, and the quantity k is related to the Newton's constant G_N,

$$k = \sqrt{8\pi G_N}. \tag{4}$$

As it has been shown in [18], the corresponding Hamiltonian for the considered linear approximation is

$$\mathcal{H} = \gamma^0 m(1 - k\,h^{00}) + \boldsymbol{\alpha}\mathbf{p} - \frac{k}{2}\{h^{00}, \boldsymbol{\alpha}\mathbf{p}\}_+ - \frac{k}{2}\{h^{ij}, \alpha^i p^j\}_+ + k\,\{\mathbf{h}, \mathbf{p}\}_+ + k\,([\nabla\times\mathbf{h}]\,\mathbf{s}), \tag{5}$$

where $\mathbf{p} = -i\hbar\boldsymbol{\nabla}$, $\boldsymbol{\alpha} = \gamma^0\boldsymbol{\gamma}$, and $\mathbf{s} = \frac{1}{2}\boldsymbol{\Sigma} = \frac{1}{2}\gamma^0\gamma^5\boldsymbol{\gamma}$. Note that in the limit of a weak gravitational field it is possible to treat gravitational perturbations $h^{oi} = \mathbf{h}$ as an external field contributing to the neutrino interaction Lagrangian. Starting from the neutrino interaction Lagrangian $\mathcal{L}$ accounting for general types of neutrino non-derivative interactions with external fields,

$$\begin{aligned} -\mathcal{L} =& g_s s(x)\bar{\nu}\nu + g_p\pi(x)\bar{\nu}\gamma^5\nu + g_v V^\mu(x)\bar{\nu}\gamma_\mu\nu + g_a A^\mu(x)\bar{\nu}\gamma_\mu\gamma^5\nu \\ &+ \frac{g_t}{2}T^{\mu\nu}\bar{\nu}\sigma_{\mu\nu}\nu + \frac{g_t'}{2}\Pi^{\mu\nu}\bar{\nu}\sigma_{\mu\nu}\gamma_5\nu, \end{aligned} \tag{6}$$

where $s, \pi, V^\mu = (V^0, \mathbf{V}), A^\mu = (A^0, \mathbf{A}), T_{\mu\nu} = (\mathbf{a}, \mathbf{b}), \Pi_{\mu\nu} = (\mathbf{c}, \mathbf{d})$ are the scalar, pseudoscalar, vector, axial-vector, tensor, pseudotensor fields, respectively, and g_i ($i = a, p, v, a, t, t'$) are the coupling constants, $\sigma_{\mu\nu} = (i/2)(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$, we get [9] the following expression for the corresponding Hamiltonian:

$$\begin{aligned} \mathcal{H} =& g_s s\rho_3 - ig_p\pi\rho_2 + g_v(V^0 - (\boldsymbol{\alpha}\mathbf{V})) - g_a(\rho_1 A^0 - (\boldsymbol{\Sigma}\mathbf{A})) \\ &- g_t(\rho_3(\boldsymbol{\Sigma}\mathbf{b}) + \rho_2(\boldsymbol{\Sigma}\mathbf{a})) - ig_t'(\rho_3(\boldsymbol{\Sigma}\mathbf{c}) - \rho_2(\boldsymbol{\Sigma}\mathbf{d})), \end{aligned} \tag{7}$$

where $\boldsymbol{\alpha} = \gamma^0\boldsymbol{\gamma},\ \rho_1 = -\gamma_5,\ \rho_3 = \gamma^0,\ \rho_2 = i\rho_1\rho_3$.

The last term in equation (5),

$$\mathcal{H}_{\text{spin}} = k(\,[\boldsymbol{\nabla} \times \mathbf{h}]\mathbf{s}), \tag{8}$$

which is responsible for the neutrino spin interaction with a gravitational field, can be identified with an axial-vector part of the general Hamiltonian given by (7) if we define the axial-vector field $A^\mu = (0, \mathbf{A})$ by the following relation

$$g_a\mathbf{A} = \frac{k}{2}[\boldsymbol{\nabla} \times \mathbf{h}], \tag{9}$$

where g_a is the coupling constant of this field to a neutrino. Following the performed in [9] derivation of the quasiclassical spin evolution equation accounting now only for the axial-vector interaction given by (9), we get the evolution equation for the neutrino spin vector $\boldsymbol{\zeta}_\nu$ in the considered gravitational field

$$\frac{d\boldsymbol{\zeta}_\nu}{dt} = \frac{2}{\gamma}[\boldsymbol{\zeta}_\nu \times \mathbf{G}], \tag{10}$$

where t is time in the laboratory frame, and, as it follows from eq.(8) of [9], the vector $\mathbf{G}$ is

$$\mathbf{G} = -\frac{k}{2}\left\{[\boldsymbol{\nabla} \times \mathbf{h}] + \frac{\gamma}{1 + 1/\gamma}\boldsymbol{\beta}(\boldsymbol{\beta}[\boldsymbol{\nabla} \times \mathbf{h}])\right\}. \tag{11}$$

Here β is the speed , $\gamma = E_\nu/m_\nu$ is the Lorentz factor of the neutrino.

It should be noted that the derived neutrino spin evolution equation (10) accounts only for the gravitational field, however the possible effects of the neutrino interactions with

the background matter and electromagnetic fields are not included here. The analogous neutrino spin evolution equation in the presence of electromagnetic fields and matter in the Minkowski space (see refs. ([6, 9, 10, 12]) is

$$\frac{d\boldsymbol{\zeta}_\nu}{dt} = \frac{2\mu}{\gamma}\Big[\boldsymbol{\zeta}_\nu \times (\mathbf{B}_0 + \mathbf{M}_0)\Big], \tag{12}$$

$$\mathbf{B}_0 = \gamma\Big(\mathbf{B}_\perp + \frac{1}{\gamma}\mathbf{B}_\parallel + \sqrt{1-\gamma^{-2}}\Big[\mathbf{E}_\perp \times \mathbf{n}\Big]\Big),\ \gamma = (1-\beta^2)^{-\frac{1}{2}}, \tag{13}$$

$$\mathbf{M_0} = \mathbf{M}_{0_\parallel} + \mathbf{M}_{0_\perp}, \tag{14}$$

$$\begin{gathered}\mathbf{M}_{0_\parallel} = \gamma\boldsymbol{\beta}\frac{n_0}{\sqrt{1-v_e^2}}\left\{\rho_e^{(1)}\left(1 - \frac{\mathbf{v}_e\boldsymbol{\beta}}{1-\gamma^{-2}}\right) - \right. \\ \left. -\rho_e^{(2)}\left(\boldsymbol{\zeta}_e\boldsymbol{\beta}\sqrt{1-v_e^2} + \frac{(\boldsymbol{\zeta}_e\mathbf{v}_e)(\boldsymbol{\beta}\mathbf{v}_e)}{1+\sqrt{1-v_e^2}}\right)\frac{1}{1-\gamma^{-2}}\right\},\end{gathered} \tag{15}$$

$$\mathbf{M}_{0_\perp} = -\frac{n_0}{\sqrt{1-v_e^2}}\left\{\mathbf{v}_{e_\perp}\left(\rho_e^{(1)} + \rho_e^{(2)}\frac{\boldsymbol{\zeta}_e\mathbf{v}_e}{1+\sqrt{1-v_e^2}}\right) + \boldsymbol{\zeta}_{e_\perp}\rho_e^{(2)}\sqrt{1-v_e^2}\right\}, \tag{16}$$

where $\mathbf{F}_\perp$ and $\mathbf{F}_\parallel$ ($\mathbf{F} = \mathbf{B}, \mathbf{E}$) are transversal and longitudinal (with respect to the direction of neutrino motion $\mathbf{n} = \boldsymbol{\beta}/\beta$) electromagnetic field components in the laboratory frame. For simplicity we neglect here the neutrino electric dipole moment, $\epsilon = 0$, and also consider the case when matter is composed of only one type of fermions (electrons). The general case of $\epsilon \neq 0$ and matter composed of different types of leptons is discussed in our papers mentioned above ([6,9,10,12]). Note that $n_0 = n_e\sqrt{1-v_e^2}$ in (15) and (16) is the invariant number density of matter given in the reference frame for which the total speed of matter is zero. The vectors $\mathbf{v}_e$, and $\boldsymbol{\zeta}_e$ ($0 \leqslant |\zeta_e|^2 \leqslant 1$) denote, respectively, the speed of the reference frame in which the mean momentum of matter (electrons) is zero, and the mean value of the polarization vector of the background electrons in the above mentioned reference frame. The coefficients $\rho_e^{(1,2)}$ are calculated if the neutrino interaction with matter is given. Therefore, within the extended standard model supplied with $SU(2)$-singlet right-handed neutrino ν_R, we have

$$\rho_e^{(1)} = \frac{\tilde{G}_F}{2\sqrt{2}\mu}, \qquad \rho_e^{(2)} = -\frac{G_F}{2\sqrt{2}\mu}, \tag{17}$$

where $\tilde{G}_F = G_F(1 + 4\sin^2\theta_W)$.

From the neutrino evolution equation (12) we have predicted that the neutrino spin precession can be induced not only by external electromagnetic fields but also by the neutrino weak interaction with particles of the background matter. Now from the neutrino spin evolution equation (10) in the gravitational field it follows that the neutrino spin procession can appear if the vector $\boldsymbol{G}$ is at least not zero. Thus, if the spin is processing the off-diagonal metric components h^{0i} have to depend on the space coordinates. The

appropriate example is provided by the Kerr geometry that corresponds to the gravitational field of a rotating with angular momentum $\boldsymbol{L}$ object. In this case

$$\mathbf{h} = \frac{k}{8\pi}[\mathbf{L} \times \mathbf{r}], \tag{18}$$

and the axial-vector field $\boldsymbol{A}$ can be written as

$$g_a \mathbf{A} = G_N \frac{3\mathbf{r}(\mathbf{L}\mathbf{r}) - r^2 \mathbf{L}}{r^5}. \tag{19}$$

Thus, we conclude that, as it follows from the neutrino spin evolution equation (10), the spin procession can appear if the neutrino is moving in a weak gravitational field of a rotating object.

3 Probability of neutrino spin oscillations in gravitational fields

Using the results of the previous section, it is possible to get the corresponding neutrino spin oscillations Hamiltonian which can be used in derivation of the neutrino spin oscillations probability in the presence of the gravitational field.

Let us consider the case when the neutrino is moving along the radial direction and decompose the vector $\mathbf{G}$ following

$$\mathbf{G} = \mathbf{G}_{\parallel} + \mathbf{G}_{\perp}, \tag{20}$$

where

$$\mathbf{G}_{\parallel} = -\gamma \frac{G_N L}{r^3} \mathbf{n} \cos\theta, \tag{21}$$

and

$$\mathbf{G}_{\perp} = \frac{1}{2} \frac{G_N L}{r^3} \mathbf{n}_{\perp} \sin\theta \tag{22}$$

are the longitudinal and transversal (with respect to the neutrino momentum) components of the vector $\mathbf{G}$, $\mathbf{n}_{\perp}$ is the unite orthogonal to $\mathbf{n}$ and laying in one plane with $\mathbf{n}$ and $\mathbf{L}$ vector. In order to get the probability of the neutrino spin oscillations in the gravitational field we introduce the unit vector $\mathbf{k}$

$$\mathbf{k} = \frac{\mathbf{G}}{G},$$

and rewrite eq.(10) in the form

$$\frac{d\boldsymbol{\zeta}_{\nu}}{dt} = \frac{\alpha}{r^3}[\boldsymbol{\zeta}_{\nu} \times \mathbf{k}], \tag{23}$$

where with the use of (20), (21), and (22) we have

$$\alpha = \frac{G_N L}{\gamma} \sqrt{4\gamma^2 \cos^2\theta + \sin^2\theta}.$$

If we expand the neutrino spin vector $\boldsymbol{\zeta}_\nu$ over the basis determined by the vector $\mathbf{G}$, so that

$$\boldsymbol{\zeta}_\nu = \boldsymbol{\zeta}_\nu^\perp + \boldsymbol{\zeta}_\nu^\parallel,$$

where $(\boldsymbol{\zeta}_\nu^\perp \mathbf{k}) = 0$ and $(\boldsymbol{\zeta}_\nu^\parallel \boldsymbol{\zeta}_\nu^\perp) = 0$, then the solution of eq.(23) in terms of $\boldsymbol{\zeta}_\nu^\parallel$, $\boldsymbol{\zeta}_x$ and $\boldsymbol{\zeta}_y$ ($\boldsymbol{\zeta}_\nu^\perp = \sqrt{\zeta_x^2 + \zeta_y^2}$), is given by

$$\begin{aligned} \zeta_x &= -\sqrt{1-\zeta_0^2}\sin\left(\frac{\alpha}{2\beta r^2}\right), \\ \zeta_y &= \sqrt{1-\zeta_0^2}\cos\left(\frac{\alpha}{2\beta r^2}\right), \\ \zeta_\nu^\parallel &= \zeta_0. \end{aligned} \tag{24}$$

Here ζ_0 is a constant determined by the initial conditions. Finally, for the probability of the neutrino oscillations in the gravitational field, described by the vector $\mathbf{G}$, we get

$$P_{\nu_L \to \nu_R}(t) = \frac{1}{2}[1 + (\boldsymbol{\zeta}_\nu \mathbf{n})], \tag{25}$$

were $\boldsymbol{\zeta}_\nu$ is the solution of eq.(23) given by (24).

4 Spin light of neutrino in gravitational fields

From our previous studies [10] of the $SL\nu$ we know that this radiation is emitted if the neutrino magnetic moment is processing, no matter what is the cause of this procession. Therefore, using the results for the $SL\nu$ in matter and electromagnetic fields and the analogy between neutrino spin evolution eqs. (10) and (12) we get for the total power of the *spin light of neutrino in the gravitational field*

$$I_{\rm gr} = \frac{16}{3}\mu^2\left[4(\mathbf{G}^2)^2 + \dot{\mathbf{G}}^2\right], \tag{26}$$

where μ is the neutrino magnetic moment.

Let us now consider the new phenomenon of the $SL\nu$ in the case of neutrinos moving in the gravitational fields of a rotating neutron star along its radial direction. From eqs. (20), (21), and (22) it is easy to see that for the ultra relativistic neutrinos (for the neutrinos from neutron stars the Lorentz factor could be of the order of $\gamma \sim 10^9$) the total power of the $SL\nu$ is maximal when neutrinos move along or opposite to the angular momentum $\mathbf{L}$:

$$I_{\rm gr} = 48\mu^2\gamma^4\frac{G_N^2 L^2}{r^8}\left[\frac{1}{1-\frac{1}{\gamma^2}} + \frac{G_N^2 L^2}{9r^4}\right]. \tag{27}$$

For a neutron star with the radius ~ 10 km we get

$$I_{\rm gr} = 3\mu^2\gamma^4 \left[1 + \frac{1}{144}\left(\frac{1\,{\rm km}}{r}\right)^4\right]\frac{1}{r^8}. \tag{28}$$

It follows that the total radiation power of the $SL\nu$ in the gravitational field is proportional to $\sim \gamma^4$ and the main contribution is given by the second term in (27) which originates from the derivative term in the general expression (26). It is also possible to show that there is a strong beaming effect and that the radiation is confined within a small cone in the direction of the neutrino propagation given by the angle $\delta\theta \sim \gamma^{-1}$. Thus, we predict that the angular distribution of the $SL\nu$ in the gravitational field of a rotating neutron star is not isotropic even if neutrinos are moving symmetrically in all radial directions from the neutron star. In this case the $SL\nu$ in the gravitational field is radiated more effectively by neutrinos moving along or against the axis of the neutron star rotation.

It is also possible to consider the combining effect of the electromagnetic, weak and gravitational fields on neutrino spin procession. From (10) and (12) we get for the neutrino spin evolution in this case

$$\frac{d\boldsymbol{\zeta}_\nu}{dt} = \frac{2}{\gamma}[\boldsymbol{\zeta}_\nu \times (\mu\mathbf{B_0} + \mu\mathbf{M_0} + \mathbf{G})], \tag{29}$$

where $\mathbf{B_0}$, $\mathbf{M_0}$ are given by eqs.(13) and (14) respectively. The gravity effect enters through the vector $\mathbf{G}$, which in the considered case of a rotating neutron star, is given by eq.(20).

It is interesting to compare the contributions to the $SL\nu$ from the neutrino interaction with the background matter and with the background gravitational field. For illustration, let us investigate the two cases: i) the $SL\nu$ from a rotating black hole produced by neutrinos moving in accretion disk, and ii) the $SL\nu$ from a quasar produced by neutrinos moving along the relativistic jet. We take matter consisting of hydrogen. In the first case we assume that matter in the accretion disk is of constant density and slowly moving along the radial direction. In the second case we consider matter of the jet moving with relativistic speed. We omit here the electromagnetic interaction but account for the neutrino couplings with matter and gravitational field, aiming to consider also the interplay between them. This modelling of the background environments, although is rather simplified, could reveal the main features of the effects under consideration.

We start with the general expression [9] for the neutrino spin evolution Hamiltonian and extract for the further consideration the terms accounting for the neutrino interactions with the gravitational field and matter:

$$\mathcal{H} = g_a(\boldsymbol{\Sigma}\mathbf{A}) + \left(\gamma_5 f^0(x) + (\boldsymbol{\Sigma}\,\mathbf{f}(x))\right), \tag{30}$$

where $g_a\mathbf{A}$ is given by (9). The matter term contribution in (30) is determined in the general form as

$$f^\mu = \frac{G_F}{\sqrt{2}}\sum_{f=e,p,n}\left(j^\mu_f q^{(1)}_f + \lambda^\mu_f q^{(2)}_f\right), \tag{31}$$

$$q_f^{(1)} = (I_{3L}^{(f)} - 2Q^{(f)} \sin^2\theta_W + \delta_{ef}), \quad q_f^{(2)} = -(I_{3L}^{(f)} + \delta_{ef}), \tag{32}$$
$$\delta_{ef} = \begin{cases} 1, & f = e, \\ 0, & f = n,\, p, \end{cases}$$

however in the considered case of matter composed of hydrogen there is no contributions from neutrons n. In this equations, $I_{3L}^{(f)}$ denotes the third component of weak isospin for the fermion f, and $Q^{(f)}$ is the electric charge of the fermion. The matter current and polarization are [8], respectively,

$$j^\mu = (n_f, n_f \mathbf{v}_f), \tag{33}$$
$$\lambda^\mu = \left(n_f \boldsymbol{\zeta}_f \mathbf{v}_f, n_f \boldsymbol{\zeta}_f \sqrt{1 - v_f^2} + \frac{n_f \mathbf{v}_f \left(\boldsymbol{\zeta}_f \mathbf{v}_f\right)}{1 + \sqrt{1 - v_f^2}} \right), f = e, p. \tag{34}$$

Finally, for the neutrino spin evolution equation in the considered case we have

$$\frac{d\boldsymbol{\zeta}_\nu}{dt} = \frac{2}{\gamma} \left[\boldsymbol{\zeta} \times (\mu \mathbf{M}_0 + \mathbf{G}) \right], \tag{35}$$

where the matter term according to eqs.(30), (31), and (32) is

$$\mathbf{M}_0 = \gamma \boldsymbol{\beta} \left(f^0 - \frac{\gamma}{1+\gamma} (\mathbf{f} \boldsymbol{\beta}) \right) - \mathbf{f}, \tag{36}$$

and the gravitational field effect is described by the vector $\mathbf{G}$ which is given by (20). Thus, for the total radiation power of the $SL\nu$ in matter and the gravitational filed we find,

$$I_{\mathrm{gr,matt}} = \frac{16}{3} \mu^2 \left[4 \big((\mu \mathbf{M}_0 + \mathbf{G})^2 \big)^2 + \big(\mu \dot{\mathbf{M}}_0 + \dot{\mathbf{G}} \big)^2 \right]. \tag{37}$$

It is evident that the total radiation power, $I_{\mathrm{gr,matt}}$, is composed of the three contributions,

$$I_{\mathrm{gr,matt}} = I_{\mathrm{gr}} + I_{\mathrm{matt}} + I_{\mathrm{gr+matt}}, \tag{38}$$

where I_{gr} is the radiation power due to the neutrino magnetic moment interaction with the gravitational field, I_{matt} is the radiation power due to the neutrino weak interaction with the background matter, and $I_{\mathrm{gr+matt}}$ stands for the interference effect of the gravitational and weak interactions. In an analogy with the neutrino spin flip in matter (see [10]) we predict the similar effect for neutrino spin in gravitational fields: the initially unpolarized neutrino beam (equal mixture of active left-handed and sterile right-handed neutrinos) can be converted to the totally polarized beam composed of only ν_R.

Let us turn to the discussion on the $SL\nu$ in the two particular cases mentioned above:

i) It is obvious that in this case at small distances r from the center of the black hole the contribution to the $SL\nu$ coming from the gravitational interaction dominates over

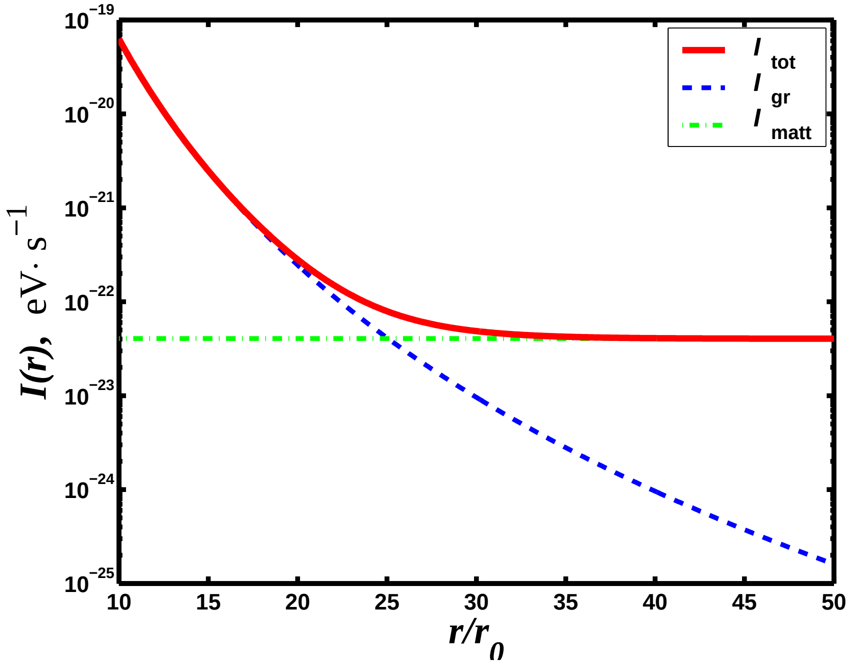

Figure 1: The total radiation power of the spin light of neutrino, $I(r) = I_{\rm gr} + I_{\rm matt} + I_{\rm gr+matt}$, versus radial distance r is shown by the red solid line. The blue dashed and green dash-dotted lines correspond, respectively, to the gravitational, $I_{\rm gr}$, and matter, $I_{\rm matt}$, contributions without account for the interference term $I_{\rm gr+matt}$.

that from the interaction with matter. As neutrino propagates away from the center, the gravity contribution fades and starting from a certain distance from the black hole center the matter interaction contribution to the $SL\nu$ becomes predominant. The total radiation power, $I_{\rm gr,matt} = I(r)$, versus the distance r is shown in Fig. 1. The green dash-dotted line corresponds to the matter contribution, while the blue dashed line stands for the gravitational field contribution to the $SL\nu$. For the neutrino gamma factor, neutrino magnetic moment and matter density we take, respectively, the following values of $\gamma = 10^{12}$, $\mu = 10^{-10}\mu_0$ and $n = 10^{24}\,\rm cm^{-3}$, where μ_0 is the Bohr magneton. It is supposed that the gravitational field is produced by the rotating object with the solar mass $M = M_\odot$, and the angular momentum is chosen to be equal to the maximal allowed value $L = r_0^2/(4G_N)$ (see, for instance, [27]) where r_0 is the Schwarzschild radius.

ii) In this case, following eq.(35) the total radiation power is

$$I_{\rm gr,matt} = \frac{64}{3}\mu^2\gamma^4\left[\left(\frac{G_N\mathbf{L}}{r^3} - \frac{G_F}{\sqrt{2}}\sum_{f=e,p} n_f(\mathbf{v}_f - \boldsymbol{\beta})\right)^4 + \frac{9}{4}\frac{G_N^2\beta^2L^2}{r^8}\right]. \tag{39}$$

If matter is moving with relativistic speed along the direction of the neutrino propagation then the matter contribution to $I_{\rm gr,matt}$ is washed out because of the presence of the term $(\mathbf{v}_f - \boldsymbol{\beta}) \sim 0$. This happens due to the particular properties of the neutrino oscillations

in moving matter (see [4, 5, 8, 9, 12]). Thus, we conclude that in the case of the neutrinos moving in the relativistic jet of a quasar the $SL\nu$ could be produced by the gravitational field, whereas there is no important contribution from interaction with matter.

5 $SL\nu$ photon energy in gravitational fields

Now we discuss the energy range of the $SL\nu$ photons emitted in a background where only the presence of the gravitational field of a rotating object generates the neutrino spin procession (it is supposed that neither matter nor electromagnetic fields of the background gives important contribution to the neutrino spin evolution). In order to get an estimation for the characteristic scale of the energy of the emitted photons we suppose that the variation of the vector $\mathbf{G}$ (which describes the influence of the gravitational field) on the neutrino travelled distance Δr is much less then the frequency of the neutrino spin precession,

$$\frac{\Delta G}{G} \ll \omega \Delta r. \tag{40}$$

If we consider the neutrino propagating along the rotation axis, $\theta = 0$, and also take into account followed from eqs.(21) and (22) relation $G_{\parallel} \gg G_{\perp}$ then the emitted photon energy in the rest frame of the neutrino is

$$\omega_0 \sim \frac{G_N L}{r^3} \gamma. \tag{41}$$

To calculate the photon energy in the laboratory reference frame we should use the well known formula

$$\omega = \omega_0 \frac{\sqrt{1-\beta^2}}{1-\beta\cos\vartheta}, \tag{42}$$

where ϑ is the angle between the neutrino speed and the direction towards the observer. Due to the fact that the $SL\nu$ is strongly beamed within a small cone there is a reason to consider the case of $\vartheta \approx 0$. Therefore, the emitted photon energy in the laboratory frame is

$$\omega \sim \omega_0 \gamma \sim \frac{G_N L}{r^3} \gamma^2. \tag{43}$$

For the set of parameters that we have just considered in the case of a rotating black hole, eq.(43) reads

$$\omega \sim 10^{-11} \times \gamma^2 \left(\frac{r_0}{r}\right)^3 \text{ eV}. \tag{44}$$

For $\gamma \sim 10^{12}$ and $r \sim 10\ r_0$ we obtain $\omega \sim 10\,\text{GeV}$. Note that for the mass of neutrino $m_\nu \sim 1\,\text{eV}$ we also obtain $(\omega/E_\nu) \sim 10^{-2} \ll 1$, i.e. the quasiclassical approach to the neutrino spin evolution is valid in this case. These properties of $SL\nu$ enable us to predict that this radiation should be important in different astrophysical environments (quasars, gamma-ray bursts etc) and in dense plasma of the early Universe.

6 Conclusion

We also should like to point out that although the predicted above and in [10] the *spin light of neutrino* in matter, electromagnetic and gravitational fields is supposed to be radiated in the process

$$\nu_1 \to \nu_2 + \gamma, \tag{45}$$

without change of the neutrino flavour state (the neutrinos in the initial and final states, ν_1 and ν_2, are of the same flavour), it is possible to generalize, with a minor modification, the corresponding equations for the case when the $SL\nu$ is radiated due to the neutrino transition magnetic moment interaction with the background fields. The latter corresponds to models of the Dirac neutrinos with non-diagonal magnetic moments or the Majorana neutrinos which could also have transitional (magnetic) moments.

In conclusion, we consider the neutrino spin evolution in presence of gravitational fields and derive for the first time the corresponding neutrino oscillations probability. We also predict the possibility of a new mechanism of the spin light of neutrino ($SL\nu$) in a gravitational field which, as we expect, could have consequences in astrophysical and cosmological settings. As examples, we analyzed the $SL\nu$ in particular cases of neutrino moving in the vicinity of a rotating neutron star, black hole and the relativistic jet of a quasar.

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